Mixed-Integer-Programming Methods for Gas Network Optimization

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joint work with Björn Geißler, Alexander Martin, Antonio Morsi

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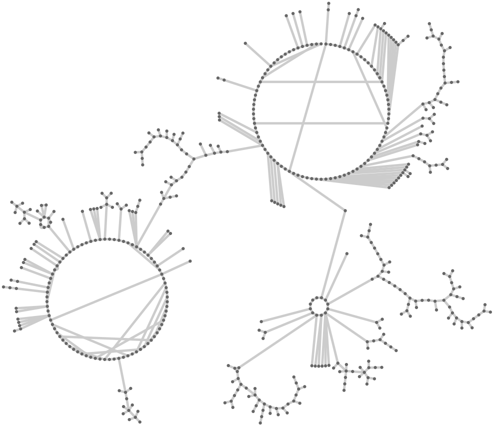
Friedrich-Alexander-Universität Erlangen-Nürnberg

Outline of the Problem



Given

- gas network
- supplies/demands
- pressure bounds

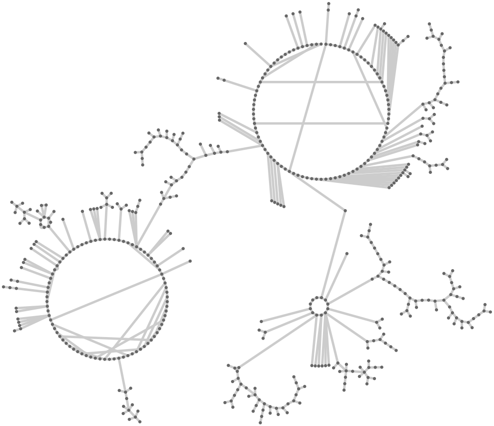
Find

network configuration that

- satisfies demands
- minimizes transport energy

Deciding feasiblity is necessary for other problems

Preliminary Results



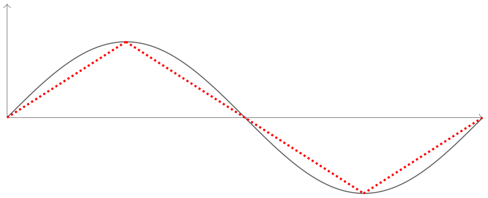
Test set:
3882 real-world instances

- feasible solution in 1 min on average
- 3862 ran less than 15 min

Feasiblity

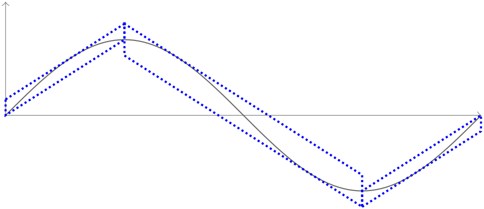
validated with NLP model developed in Hannover

Our Approach



- replace non-linear functions by piecewise-linear functions
- solve the resulting MIP

Our Approach



- replace non-linear functions by piecewise-linear sets
- solve the resulting MIP

Structure of the Project

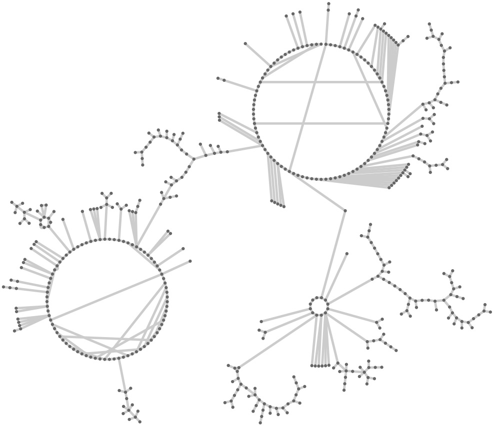
Funding

- Open Grid Europe
- German Federal Government

Partners

- FAU Erlangen-Nuremberg
- Humboldt University Berlin
- Leibniz University Hannover
- TU Braunschweig
- University Duisburg-Essen
- Weierstraß Institute Berlin
- Zuse Institute Berlin

A Gas Network



Network data

- 518 nodes / 544 edges
- 130 nodes with non-zero supply/demand
- ≈ 30 valves
- ≈ 5 compressor stations
- ≈ 10 resistors
- ≈ 20 “control valves”

Smallest relevant network in the project

Problems

Standard contract

"On any given day you are allowed to supply/demand up to X units of gas at node v if you have matching partners at some other nodes"

Problems

- Given a supply/demand situation on a given day, is it technically feasible?
- How large may X be?
- Given a set of nodes, how large may we choose each X_v such they can be satisfied simultaneously?
- If one of the above problems has no satisfying solution, where can we build a network extension to ameliorate the situation?

Modeling – Discrete and Nonlinear

Nonlinear

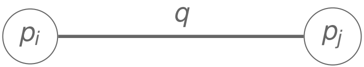
- Pipes
- Compressors
- Resistors

Discrete

- Valves
- “Control Valves”
- Switching inside compressor stations

Modeling – Pressure-loss in Pipes

$$p_j^2 = \left(p_i^2 - \Lambda |q| q \frac{e^S - 1}{S}\right) e^{-S}$$



Λ computed from gas and pipe parameters

S computed from height difference of endpoints

Modeling – Power used by Compressors

$$P = \Omega \left(\left(\frac{p_j}{p_i} \right)^{\gamma} - 1 \right) q$$

γ computed from gas parameters

Ω computed from compressor data

Missing

- (nonlinear) description of feasible set

Solution approaches

$$p_j^2 = \left(p_i^2 - \Lambda |q| q \frac{e^S - 1}{S}\right) e^{-S}$$

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- Rewrite the function, preferably as 1D-functions
- Compute the piecewise linearization

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Solution approaches

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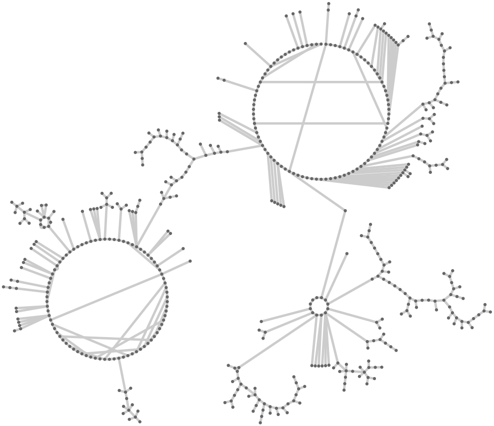
$$X_1 = \hat{p}_j \ (:= p_j^2)$$
$$X_2 = \hat{p}_i \ (:= p_i^2)$$
$$X_3 = |q|q$$
$$\alpha = e^{-S}$$
$$\beta = e^{-S} \Lambda \frac{e^S - 1}{S}$$
$$X_1 = \alpha X_2 - \beta X_3$$

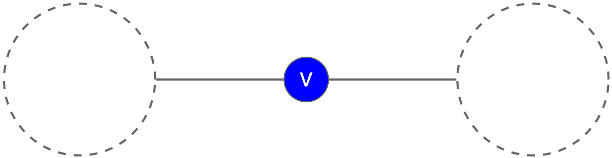
- Rewrite the function, preferably as 1D-functions
- Compute the piecewise linearization
- Often values are not needed

Experimental – Decomposition



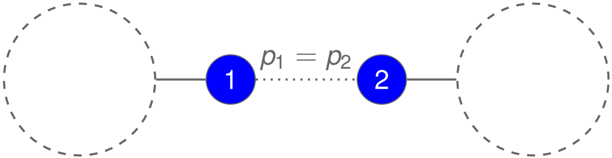
- Loosely coupled network
- Components easy to solve in isolation
- Larger networks are denser

Experimental – Langrangian relaxation



- Split off trees and 2-connected components
- Langrangian relaxation: relax coupling conditions
- Solve relaxation using bundle method
- Try to get a feasible solution by rounding
- (use SPQR-tree for large 2-connected components)

Experimental – Langrangian relaxation



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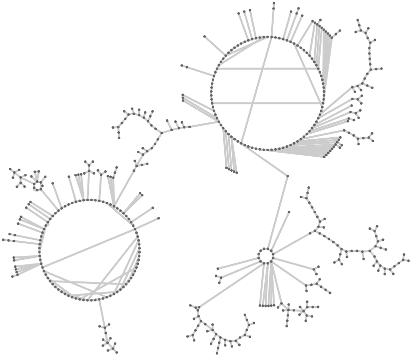
Preliminary Results



Results on the small network

- Testset: 3882 instances
- 3862 feasible in 15 minutes
- Average time per instance: less than 1 minute
- Validated with NLP model

Preliminary Results

Results on a large network

- Testset: 1128 instances
- 728 feasible within 2 hours
- 224 infeasible
- 176 no solution after 2 hours
- Not all solutions pass NLP test

DISCRETE OPTIMIZATION

Thank you!