# A traveling salesman problem with quadratic cost structure

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# Outline

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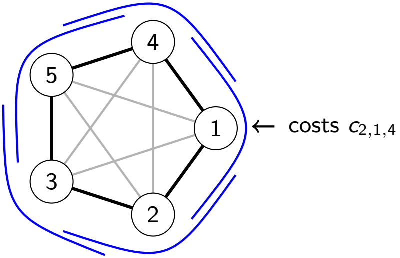
# Problem description of **QSTSP**

**Given:**

- undirected complete 2-graph $G = (V, E)$, $V = \{1, \dots, n\}$,
  $V^2 := \{\{i, j\} \colon i, j \in V, i \neq j\}$ (write $ij$) – arcs
  $V^3 := \{\langle i, j, k\rangle = \langle k, j, i\rangle \colon i, j, k \in V, |\{i, j, k\}| = 3\}$ (write $ijk$) – 2-arcs
- cost function $c \colon V^3 \to \mathbb{R}_+$ with $c_{ijk}$ – costs of path $i - j - k$,
  $\Rightarrow$ quadratic cost structure

**Goal:** find tour $T = (i_1, \dots, i_n, i_1)$ minimizing

$$\sum_{k=1}^{n-2} c_{i_k i_{k+1} i_{k+2}} + c_{i_{n-1} i_n i_1} + c_{i_n i_1 i_2}$$



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**Special case of QSTSP** Angular-Metric TSP, see Aggarwal et al. (1997)
given points in the plane – find tour minimizing total direction changes
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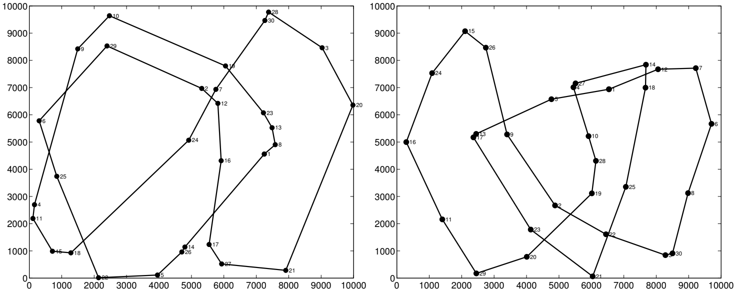

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**Complexity** NP-complete, even the corresponding cycle cover problem is NP-complete

# Outline

# Integer Linear Program

Linearization of the following quadratic integer model

$$\min \sum_{ijk \in V^3} c_{ijk}\, x_{ij} x_{jk}$$

$$\text{s.t.} \sum_{ij \in V^2} x_{ij} = 2, \qquad i \in V \qquad \text{(degree)}$$

$$\sum_{\substack{ij \in V^2 \\ i \in S, j \in V \setminus S}} x_{ij} \geq 2, \qquad S \subset V, 2 \leq |S| \leq n-2 \qquad \text{(subtour)}$$

$$x_{ij} \in \{0,1\}, \qquad ij \in V^2$$

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$$x_{ij} \in \{0,1\}, \qquad ij \in V^2$$

$$x_{ij} = \sum_{ijk \in V^3} y_{ijk}, \qquad ij \in V^2 \qquad \text{(flow)}$$

$$x_{ij} = \sum_{kij \in V^3} y_{kij}, \qquad ij \in V^2 \qquad \text{(flow)}$$

$$y_{ijk} \in [0,1], \qquad ijk \in V^3$$

# Dimension of the polytope $P_{QSTSP}$

variables: $3\binom{n}{3} + \binom{n}{2}$

equality constraints: $n + 2\binom{n}{2} = n^2$

## Observation

The constraint matrix of the QSTSP (degree, flow) has full row rank.

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Regard the Quadratic Symmetric Cycle Cover Polytope $P_{QSCCP_n}$ (subtours are allowed).

## Lemma

*The dimension of $P_{QSCCP_n}$ equals $3\binom{n}{3} + \binom{n}{2} - n^2$ for $n \geq 7$.*

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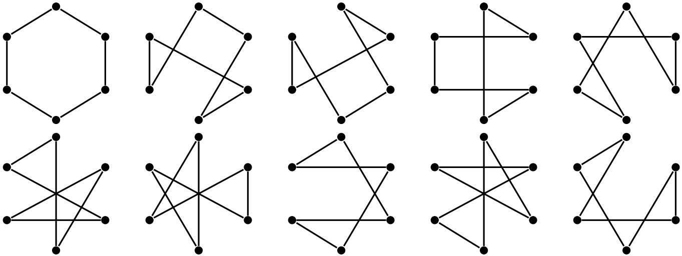
For the STSP: Grötschel and Padberg used an arc-disjoint Hamiltonian cycle decomposition of the complete graph $G_n$.

# Dimension of the polytope $P_{QSTSP}$

## Question

Is there a 2-arc-disjoint Hamiltonian. cycle decomposition of the complete 2-graph $G_n$, $n \geq 3$?



Related to an open question (Bailey, Stevens) concerning the decomposition of complete uniform hypergraphs into arc-disjoint Hamiltonian cycles.

# Dimension of the polytope $P_{QATSP}$ resp. $P_{QACCP_n}$

$$\sum_{(j,i)\in V^2} x_{(j,i)} = \sum_{(i,j)\in V^2} x_{(i,j)} = 1, \qquad i \in V,$$

$$x_{(i,j)} = \sum_{(i,j,k)\in V^3} y_{(i,j,k)} = \sum_{(k,i,j)\in V^3} y_{(k,i,j)}, \qquad (i,j) \in V^2,$$

$$\sum_{\substack{(i,j)\in V^2:\\ i\in S, j\in V\setminus S}} x_{(i,j)} \geq 1, \qquad S \subset V, 1 \leq |S| \leq n-1,$$

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$V^2 = \{(i,j)\colon i,j \in V, i \neq j\}$, $V^3 = \{(i,j,k)\colon i,j,k \in V, |\{i,j,k\}| = 3\}$

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**Lemma**

*The dimension of $P_{QACCP_n}$ equals $\underbrace{n(n-1)^2}_{\#\text{ variables}} - \underbrace{(2n^2-1-n)}_{\text{rank of constr. matrix}}$ for $n \geq 7$.*

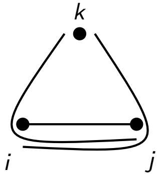
**Conjecture**

The dimension of $P_{QATSP_n}$ equals $n(n-1)^2 - (2n^2-1-n)$ for $n \geq 7$.

# Outline

# Valid inequalities

- forbid certain triangles




$$y_{kij} + y_{ijk} \leq x_{ij}$$
$$x_{ij} + x_{ik} + x_{jk} \leq 2 \;\Big|\; \cdot x_{ij}$$

lift the constraint

strengthen triangle inequalities of *Boolean Quadratic Polytope* (Padberg, 1989)

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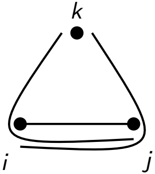


$y_{kij} + y_{ijk} \leq x_{ij}$

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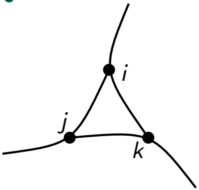


Equivalent to:
$x_{ij} + x_{ik} + x_{jk} - y_{ijk} - y_{ikj} - y_{jik} \leq 1$
see triangle inequalities of Boolean Quadratic Polytope

# conflicting arcs inequalities

At most one of the following variables can be 1:

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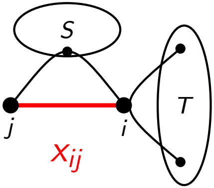

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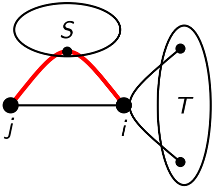
$y_{ikj}, k \in S$



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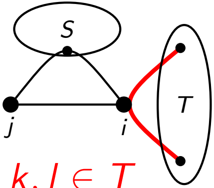
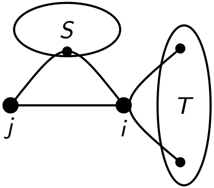



$y_{kil}, k, l \in T$

# conflicting arcs inequalities

$$x_{ij} + \sum_{\substack{ikj \in V^3 \\ k \in S}} y_{ikj} + \sum_{\substack{lim \in V^3 \\ l,m \in T}} y_{lim} \leq 1$$

for all $i, j \in V, i \neq j$, and $S, T \subset V \setminus \{i, j\}, S \cap T = \emptyset$




## Lemma

*The conflicting arcs inequalities can be separated in polynomial time.*

# conflicting arcs inequalities

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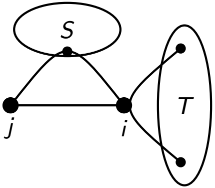


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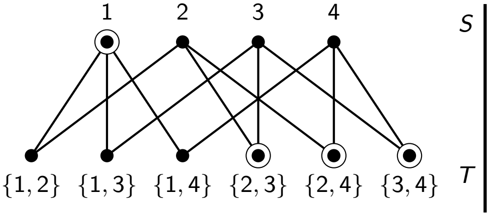
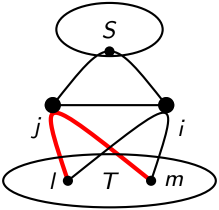
**Proof:** Transformation to *Maximal Weight Independent Set Problem* in bipartite graphs.




$n = 6, i = 5, j = 6$ :
$S = \{1\}, T = \{2, 3, 4\}$

# strengthened conflicting arcs constraints

Case $|T| = 2$
At most one of the following variables can be 1:



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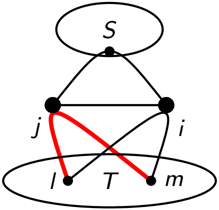


$$x_{ij} + \sum_{\substack{ikj \in V^3: \\ k \in S}} y_{ikj} + y_{lim} + y_{ljm} \leq 1$$

for all $i, j \in V, i \neq j$, and $S, T \subset V \setminus \{i, j\}, S \cap T = \emptyset, T = \{l, m\}$

# Subtour elimination constraints

$$\sum_{\substack{ij \in V^2: \\ i \in S, j \in V \setminus S}} x_{ij} \geq 2, \qquad \text{for all } S \subset V, 2 \leq |S| \leq n-2$$

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Using only $y$-variables

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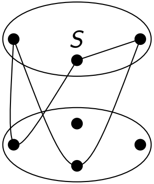
$$\forall S \subset V, 2 \leq |S| < \frac{n}{2}:$$

$$\sum_{\substack{ijk \in V^3: \\ i \in S, j,k \in V \setminus S}} y_{i,j,k} \geq 2, \qquad (1)$$



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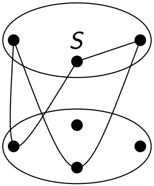


**Lemma**

*The separation problem for inequalities* (1) *is* **NP**-*complete.*

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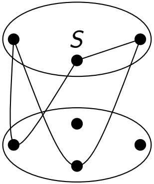


Case $|S| \geq \frac{n}{2}$:

$$\sum_{\substack{ij \in V^2: \\ i \in S, j \in V \setminus S}} x_{ij} - 2 \sum_{\substack{ikj \in V^3: \\ i,j \in S, k \in T}} y_{i,k,j} \geq 2, \quad \text{for all } S, T \subset V, S \cap T = \emptyset, |S| + |T| = n-1$$

# Outline

# Semidefinite relaxation

semidefinite relaxation bases on QIP:

$$\min \sum_{ijk \in V^3} c_{i,j,k} \cdot x_{ij} x_{jk}$$

$$\text{s.t. } x \in \mathbf{TSP}(n) \qquad \text{(TSP-polytope)}$$

with $x = (x_{12}, x_{13}, \ldots, x_{n-1,n})^T$

Construction of rank-one matrix: $X = \begin{pmatrix} 1 \\ x \end{pmatrix} \begin{pmatrix} 1 \\ x \end{pmatrix}^T$

Notation: $x_{ij,kl} \mathrel{\hat{=}} x_{ij} \cdot x_{kl}$

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$$X_{1,1} = 1$$

$$X_{1,i} = X_{i,i}, \forall\, i = 2, \dots, n+1 \qquad (x_{ij} = x_{ij,ij})$$

$$0 \leq X \leq E$$

$$\text{rank}(X) = 1$$

$$X \succeq 0$$

# Semidefinite relaxation

$$
\begin{aligned}
\min \ & \sum_{ijk \in V^3} c_{ijk} x_{ij,jk} & \\
\text{s.t.} & \sum_{ij \in V^2} x_{ij} = 2, & i \in V \\
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$$0 \leq X \leq E$$

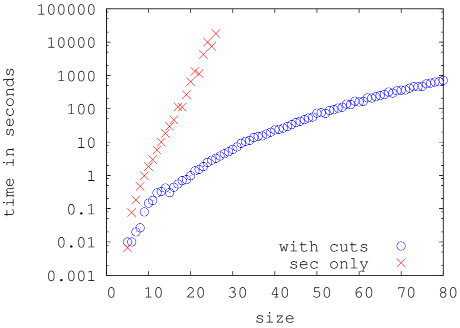
$$X \succeq 0$$

strengthening: presented cuts, inequalities of the Boolean Quadratic Polytope

# Outline

# Computational results – real-world instances

- 3 instances $5 \leq n \leq 80$
- # variables for $n = 80$: $\sim 500000$
- all instances solved in $< 13$ minutes with the presented cuts
- only SEC: running times much higher



# Computational results – random instances

10 instances

uni random asymmetric: $c_{ijk}$ uniformly at random in $\{0, 1, \ldots, 10000\}$

random angular: points $i \in V$ uniformly at random in $\{0, 1, \ldots, 10000\}^2$,

$$c_{ijk} = \left\lfloor \frac{18000}{\pi} \arccos\left( \left( \frac{v_j - v_i}{\|v_j - v_i\|} \right)^T \left( \frac{v_k - v_j}{\|v_k - v_j\|} \right) \right) \right\rfloor$$

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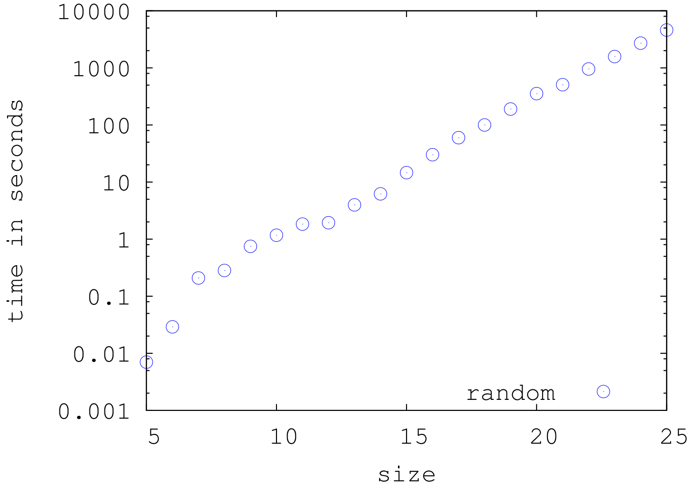
**Computer:** Intel Core i7 CPU 920, 2.67 GHz, 12 GB RAM

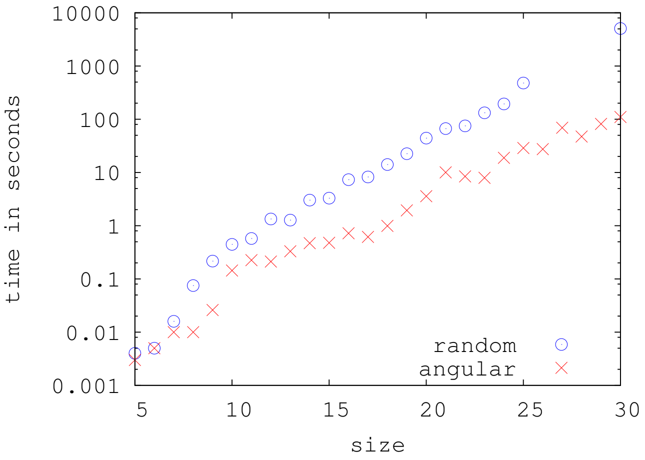
- **(MIP)**: use SCIP, LP with CPLEX 12.1,
- **(SDP)**: use Matlab and SDPT3

# Computational results – asymmetric random instances



# Computational results – symmetric instances

random and random angular instances




But: for random instances better not to separate additional inequalities

# Computational results – symmetric random instances

Comparison of the value of the gaps $[(opt - relax)/relax] \cdot 100\%$ at the root node

IP root relaxation of IP

SDP1 SDP relaxation, all inequalities only on the $y$-support

SDP2 additional $x_{ij,kl} \geq 0$ for all matrix entries

SDP3 additional triangle inequalities on whole matrix

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| $n$ | IP | SDP1 | SDP2 | SDP3 |
|---|---|---|---|---|
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7 | 0.00 | 0.00 | 0.00 | 0.00 |
| 8 | 1.74 | 0.43 | 0.30 | 0.00 |
| 9 | 2.73 | 1.09 | 0.69 | 0.02 |
| 10 | 10.35 | 4.79 | 2.99 | 0.76 |
| 11 | 13.58 | 8.30 | 5.21 | 2.63 |
| 12 | 18.60 | 11.77 | 8.38 | 5.32 |
| 13 | 19.05 | 10.93 | 6.79 | 4.17 |
| 14 | 23.18 | 14.55 | 9.94 | 7.48 |
| 15 | 23.61 | 13.31 | 8.50 | 6.45 |

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| $n$ | IP |
|---|---|
| 16 | 35.21 |
| 17 | 31.69 |
| 18 | 36.53 |
| 19 | 40.11 |
| 20 | 44.80 |
| 21 | 47.36 |
| 22 | 41.65 |
| 23 | 46.44 |
| 24 | 42.75 |
| 25 | 51.03 |
| 30 | 60.14 |

# Outline

# Further work

- further polyhedral studies, includes dimension of the corresponding polytopes
- separation heuristics for extended SEC
- use SDP-bounds for Branch-and-Cut
- use spectral bundle method for SDP
  - Which support extension is useful?
  - Which (in-)equalities should be used?
- extend neighborhood

# Thank you for your attention.
# Questions?

The Cluster of Excellence “Energy-Efficient Product and Process Innovation in Production Engineering”(eniPROD®) is funded by the European Union (European Regional Development Fund) and the Free State of Saxony.

Europa fördert Sachsen.
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