Multi-objective separation for sequentially coordinated cutting plane generation

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Cutting plane methods in integer linear programming

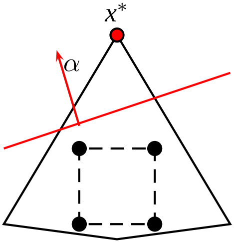
Given $A \in \mathbb{Q}^{m \times n}$, c and $b \in \mathbb{Q}^n$, consider the Integer Program

$$
\begin{array}{lll}
(P) & \min & cx \\
 & s.t. & Ax \leq b \\
 & & x \in \mathbb{Z}^{n+}
\end{array}
$$

Relaxation obtained by removing constraints or relaxing integrality

Usual cutting plane loop

1. Find optimal solution x^* of relaxation
2. Generate one or more cuts $\alpha x \leq \alpha_0, (\alpha, \alpha_0) \in \mathcal{C}$ which are maximally violated
3. Add cut(s)



General goal: Try to improve the efficiency of pure cutting plane methods by adopting a different cut generation scheme

Reducing CPU time or number of cuts/rounds to close a certain gap

Outline

1) How cutting planes are generated in practice (cut selection procedures, cut quality measures)

2) Propose a scheme based on sequentially coordinated cutting plane generation

3) Experiments on two combinatorial problems (Max Clique, Min Steiner tree)

Current practice

Two-phase approach

1) Generate many cuts, usually via separation problem where cut violation is maximized (often heuristically)

2) Select a subset (<u>cut selection procedure</u>) based on <u>cut quality measures</u>

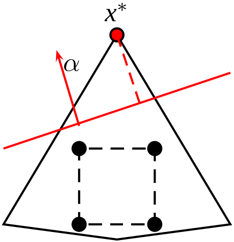
Literature

[BalasCeriaCornuejols1993]
[BalasCeriaCornuejols1996]
[BalasCeriaCornuejolsNatraj1996]
[FischettiLodi2007]
[AndreelloCapraraFischetti2007]
[DashGunlukLodi2010]
[BalasSaxena2008]
[BonamiCornuejolsDashFischettiLodi2008]
[FischettiLodi2007b]
[BalasFischettiZanette2009]
[CookFukusawaGoycoolea2006]
[Achterberg2009]
[WesselmannSuhl2010]

Cut quality measures

Distance

- Violation: $\alpha x^* - \alpha_0$ (algebraic distance)
- Depth: $\dfrac{\alpha x^* - \alpha_0}{\|\alpha\|_2}$ (geometric distance)
- Rotated steepness [CookFukusawaGoycoolea2006]
- Variants [WesselmannSuhl2010]



Bound variation

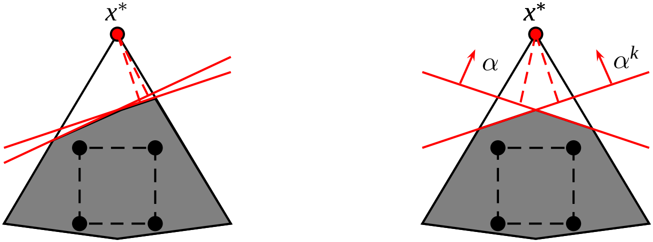
- Exact
- Approximated

Density/sparsity

- Number of nonzero components of α

- Cut parallelism: cosine of angle between α and α^k of a previous cut

$$\frac{\alpha \alpha^k}{\|\alpha\|_2 \, \|\alpha^k\|_2}$$



- Objective function parallelism: cosine of angle between α and c

Cut selection

Usual procedure

After having generated many cuts

1) order w.r.t. violation/depth
2) add w.r.t. order, only if not too parallel w.r.t. previous cuts

Extension: Order w.r.t. weighted sum of depth, cut parallelism, objective function parallelism [Achterberg2009]

Observation: By disfavoring almost parallel cuts we tend to favor a kind of cut diversity

Cut diversity in the literature

- In [FischettiLodi2007a] for Chvàtal-Gomory Closure (via MIP)
  Removing UBs on CG multipliers ($u < 1$) $\rightarrow$ larger gap closed (in 100 rounds)
  Larger feasible set $\rightarrow$ CPLEX finds more diverse solutions

- In [BalasSaxena2008] for Split Closure (parametric MIP on a grid)
  Ad hoc method to enforce set of orthogonal disjunctions

- In [BalasFischettiZanette2008] for Fractional Gomory cuts
  Reoptimization via lexicographic simplex $\rightarrow$ diversified sequence of x^*

Our contribution

General goal: Try to improve the efficiency of pure cutting plane methods by adopting a different cut generation scheme

Idea: Consider cut quality measures directly when generating a cut
Focus: cut diversity

Main issues

1) Choice of cut quality measures (not too hard to optimize)
2) Devise an "efficiently solvable" multi-objective separation problem

Setting

- Cuts with 0-1 coefficients ($\alpha \in \{0,1\}^n$), including cut, clique, stable set, cycle, cover inequalities
- For simplicity: unit right-hand side ($\alpha_0 = 1$)

Issue #1: Choice of cut quality measures

To somehow mimick a cut selection procedure, we need

1) a measure of distance
2) a measure of diversity

1) Measure of distance

Cut violation

- linear (easier to optimize)
- avoids drawbacks of cut depth $\frac{\alpha x^* - 1}{\|\alpha\|_2}$ related to cut domination

Observation: Depth trades violation for sparsity: good for $\alpha x \geq 1$, bad for $\alpha x \leq 1$

Example

		violation	$\|\alpha\|_2$	depth
$x^* =$	$(1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{2})$			
$\alpha^1 =$	$(1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$	1	$\sqrt{2}$	$\frac{1}{\sqrt{2}} = 0.57$
$\alpha^2 =$	$(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$	1.5	$\sqrt{9}$	$\frac{1.5}{\sqrt{9}} = 0.5$

Depth favors a cut which is stronger if $\alpha x \geq 1$ but weaker if $\alpha x \leq 1$

2) Measure of diversity

Given the current cut $\alpha x \leq 1$ and a previous cut $\alpha^k x \leq 1$,

consider 1-norm distance between normal vectors: $\left\|\alpha - \alpha^k\right\|_1 = \sum_{i=1}^{n} |\alpha_i - \alpha_i^k|$

Property: Linear for cuts with binary coefficients: $\left\|\alpha - \alpha^k\right\|_1 = e\alpha - 2\alpha\alpha^k + e\alpha^k$ where e all-one vector

Our choice: 1-norm distance between α and a weighted combination $\bar{\alpha}^t$ of the normal vectors of the previous cuts

$$\|\alpha - \bar{\alpha}^t\|_1$$

Natural options:

- previous cut $\bar{\alpha}^t := \alpha^t$ (too miopic)
- arithmetic average $\bar{\alpha}^t := \frac{1}{t}\sum_{k=1}^{t} \alpha^k$
- weighted average $\bar{\alpha}^t := \bar{\alpha}^{t-1}\lambda + \alpha^t(1-\lambda)$

Issue # 2: Multi-objective separation problem

In many cases multiplicity of maximally violated cuts

Idea: Among all the maximally violated cuts generate a cut which is maximally diverse w.r.t. all the previous cuts $(\alpha^1, 1), \ldots, (\alpha^t, 1)$

Since each cut depends on the sequence of previous cuts this introduces a form of cut coordination – sequentially coordinated cutting plane generation

Sequentially-Coordinated Separation

$$\begin{array}{rc} \max & \|\alpha - \bar{\alpha}^t\|_1 \\ s.t. & \alpha = \text{argmax}\{\alpha x^* - 1\} \\ & (\alpha, 1) \in \mathcal{C} \end{array}$$

bi-objective with priority

For an appropriate $\epsilon > 0$, we can cast it as a single objective problem

$$\begin{array}{ll} \max & \alpha x^* + \epsilon \|\alpha - \bar{\alpha}^t\|_1 = \alpha x^* + \epsilon(e - 2\bar{\alpha}^t)\alpha + [e\alpha^t] \\ s.t. & (\alpha, 1) \in \mathcal{C} \end{array}$$

Additional term $\epsilon(e - 2\bar{\alpha}^t)_i$

- strictly positive if $\bar{\alpha}^t_i < \frac{1}{2}$ favors letting $\alpha_j = 1$
- strictly negative if $\bar{\alpha}^t_i > \frac{1}{2}$ favors letting $\alpha_j = 0$

Sequentially-Coordinated Separation amounts to Standard Separation with a different objective function vector $\hat{x} := x^* + \epsilon(e - 2\bar{\alpha}^t)$ (unrestricted in sign)

Choice of ϵ

Sufficient condition: Given $f_1 = \alpha x^*, f_2 = \epsilon(e - 2\bar{\alpha}^t)\alpha$, choose ϵ such that

$$\epsilon(\max f_2 - \min f_2) < \Delta_1$$

where Δ_1 is the smallest nonzero variation in $f_1 = \alpha x^*$

Finding Δ_1 can be reduced to a Min Subset Sum problem

A lower bound on Δ_1 suffices

- General case: since $x^* \in \mathbb{Q}$, $LB = 1$ / greatest common denominator
  Ex: $x^* = (\frac{2}{3}, \frac{1}{6}, \frac{1}{9})$, $\Delta_1 = \frac{1}{3}$
- On a computer: x^* is a fixed precision (truncated) floating point vector
  LB = 10^{-p} where p position of the least significant digit (over all components)
- In pratice: truncate x^* to reasonable precision

Avoiding the generation of dominated cuts

If $x^* = (\cdot \cdot 0 \cdot \cdot)$, same violation for $\alpha' = (\cdot \cdot 1 \cdot \cdot)$ and $\alpha'' = (\cdot \cdot 0 \cdot \cdot)$
For $\alpha x \leq 1$: $\alpha' = (\cdot \cdot 1 \cdot \cdot)$ dominates $\alpha'' = (\cdot \cdot 0 \cdot \cdot)$
Cut $\alpha x \leq 1$ is undominated if α is "maximal"

Observation: If $x^* > 0$: every maximally violated cut will be undominated

Revised Standard Separation: $\max \{\alpha(x^* + \epsilon e) : (\alpha, 1) \in \mathcal{C}\}$ [KochMartin1998]

Revised Sequentially-Coordinated Separation

$$\begin{array}{lll} \max & \alpha x^* + \epsilon \|\alpha - \alpha^t\|_1 + 2\epsilon \|\alpha\|_1 = & \alpha x^* + \epsilon(3e - 2\bar{\alpha}^t)\alpha + [e\alpha^t] \\ s.t. & (\alpha, 1) \in \mathcal{C} & \end{array}$$

Additional term $\epsilon(3e - 2\bar{\alpha}^t)$

- between 2 and 3 if $\bar{\alpha}_i^t < \frac{1}{2}$ favors letting $\alpha_j = 1$
- between 1 and 2 if $\bar{\alpha}_i^t > \frac{1}{2}$ still favors letting $\alpha_j = 1$, but not as much

Revised Sequentially-Coordinated Separation amounts to the standard separation with a different nonnegative objective function vector $\hat{x} := x^* + \epsilon(3e - 2\bar{\alpha}^t)$

Revised Standard and Sequentially-Coordinated Separation: solvable with the same algorithm

Computational experiments

Setting: Pure cutting plane algorithm

Two combinatorial optimization problems

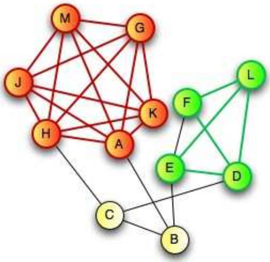
- Max Clique
- Min Steiner Tree

Single family of valid inequalities

Max Clique

Given $G = (V, E)$, find a maximum clique of G

Relaxation



$$\begin{array}{lll} \max & \sum_{i \in V} x_i & \\ s.t. & \sum_{i \in S} x_i \leq 1 & \text{for } S \in \mathcal{S} \\ & 0 \leq x_{ij} \leq 1 & \text{for } \{i,j\} \in E \end{array}$$

where $\mathcal{S}$ is the set of all maximal stable sets of G

Maximal stable set $S \Rightarrow \sum_{i \in S} x_i \leq 1$ is undominated and facet-defining

Image from *http://scienceblogs.com/goodmath/2007/07/cliques_subgraphs_and_a_bit_of.php*

Separation problem: Max Weighted Stable Set

$$\begin{array}{lll} \max & \sum_{i \in V} x_i^* \alpha_i & \\ s.t. & \alpha_i + \alpha_j \leq 1 & \text{for } \{i,j\} \notin E \\ & \alpha_i \in \{0,1\} & \text{for } \{i,j\} \in E \end{array}$$

with $\alpha \in \{0,1\}^n$ incidence vector of a stable set

Note: $\mathcal{NP}$-hard separation $\Rightarrow$ $\mathcal{NP}$-hard relaxation [GrotschelLovaszSchrijver1981]

Experiments

- One cut per round
- Instances: 24 second DIMACS Challenge
- Relaxation: CPLEX 12.2
- Separation: MIP, CPLEX 12.2 (sequential, `Threads=1, EpAGap=EpGap=EpInt=0, NumericalEmphasis=1`)

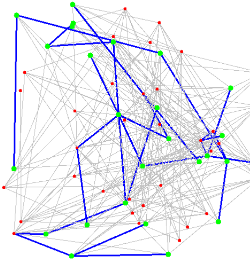
Min Steiner Tree

Given $G = (V, E)$, terminals $T \subset V$, costs $c : E \to \mathbb{R}^+$, find a Steiner tree of G

Stronger formulation for directed $G' = (V, A)$ with arbitrary root $r \in T$

Relaxation

$$
\begin{array}{lll}
\min & \displaystyle\sum_{(ij)\in A} c_{ij}x_{ij} & \\
\text{s.t.} & \displaystyle\sum_{(ij)\in\delta^+(S)} x_{ij} \geq 1 & \text{for } S \subset V : r \in S, V \setminus S \cap T \neq \emptyset \\
& 0 \leq x_{ij} \leq 1 & \text{for } (i,j) \in A
\end{array}
$$



where $\mathcal{S}$ is the set of all minimal s-t cut sets of G', $s, t \in T$

Minimal cut set $S \Rightarrow \displaystyle\sum_{(ij)\in\delta^+(S)} x_{ij} \geq 1$ undominated and facet-defining

Image from *http://morihit.net/ezxdisp/screenshots.html*

Separation problem: Min $r - t$ cut set for each terminal $t \in T \setminus \{r\}$

Polynomially solvable for nonnegative arc weights (Max Flow) $\Rightarrow$ polynomially solvable relaxation

Experiments

- Cuts in rounds (one per terminal)
- Diversity w.r.t. all previous cuts (even within same round)
- Instances: SteinLib datasets B C D (43) I640 PUC (31) –kept those solvable in 1 GB RAM
- Relaxation: CPLEX 12.2
- Separation: Edmonds-Karp (Boost Graph Library)

Computational experiments

We compare Revised Sequentially-Coordinated Separation to Revised Standard Separation

Report geometric mean of ratios

Timelimit: 1000 seconds

Machine: Dell Poweredge, Quad Core Xeon 2.0 Ghz, 4 GB RAM

Max Clique

	Revised Standard Sep.			Revised Seq-Coord. Sep.		
	Time	Cuts	Cond	Time	Cuts	Cond
c-fat200-1	2.27	194	2.7E+3	**1.57**	**152**	**2.0E+3**
c-fat200-2	0.87	75	2.3E+2	**0.56**	**45**	**4.0E+0**
c-fat200-5	7.92	265	8.5E+2	**5.82**	**205**	**3.7E+2**
c-fat500-10	68.74	412	1.1E+3	**43.27**	**249**	**4.0E+0**
c-fat500-1	15.8	348	6.0E+3	**0.51**	**27**	**6.0E+1**
c-fat500-2	52.99	523	1.3E+4	**7.98**	**184**	**1.1E+4**
c-fat500-5	21.07	256	3.1E+3	**9.8**	**127**	**3.5E+2**
hamming6-2	1.84	108	5.4E+1	**1.09**	**57**	**4.4E+1**
hamming6-4	5.98	59	3.8E+2	**4.68**	**53**	**2.6E+2**
hamming8-2	239.98	550	**1.2E+2**	**118.42**	**247**	1.2E+2
johnson16-2-4	12.61	**21**	**9.5E+0**	**11.5**	**21**	3.0E+1
johnson8-2-4	**0.04**	**11**	**2.3E+0**	0.05	**11**	1.8E+1
johnson8-4-4	**4.71**	**56**	**2.0E+2**	5.72	60	6.9E+2
MANN_a9	**1.2**	**49**	**4.1E+1**	1.45	**49**	7.2E+1
myciel3	**0.05**	15	**1.0E+1**	**0.05**	**14**	3.3E+1
myciel4	0.98	34	**9.9E+1**	**0.84**	**32**	1.1E+2
myciel5	12.93	71	6.1E+2	**8.95**	**64**	**4.6E+2**
queen10_10	**51.99**	**335**	**2.7E+3**	54.7	337	5.0E+3
queen11_11	**54.2**	**355**	**3.2E+3**	58.88	369	3.5E+3
queen12_12	68.83	395	4.8E+3	**64.67**	**393**	**4.0E+3**
queen13_13	93.01	465	5.5E+3	**84.26**	**449**	**4.1E+3**
queen14_14	**134.98**	**518**	**3.8E+3**	135.2	539	1.1E+4
queen15_15	**197.18**	592	7.7E+3	218.08	**591**	**4.8E+3**
queen16_16	301.49	678	1.1E+4	**275.97**	**656**	**5.6E+3**
Statistics	1	1	1	**0.68**	**0.73**	**0.39**

Reduction of 32% time, 27% cuts, 61% condition number

Min Steiner Tree - instances: B, C, D

	Revised Standard Separation						Revised Sequentially-Coordinated Separation					
	Time	Rnds	Cuts	Dupl	Cond	SepTime	Time	Rnds	Cuts	Dupl	Cond	SepTime
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
b10	**0.05**	16	98	12	5.5E+1	**0.01**	**0.05**	**14**	**83**	**9**	**4.5E+1**	**0.01**
b11	0.1	26	219	46	**3.8E+2**	**0.01**	**0.09**	**17**	**156**	**32**	9.1E+1	**0.01**
b12	**0.16**	**17**	375	97	3.1E+2	**0.01**	0.19	18	**257**	**18**	**3.0E+2**	**0.01**
b13	**0.04**	**11**	125	34	**3.1E+1**	**0.01**	**0.04**	**11**	**105**	**1**	5.5E+1	**0.01**
b14	**0.1**	22	379	218	2.2E+2	**0.01**	**0.1**	**15**	**227**	**11**	**1.3E+2**	**0.01**
b15	0.08	10	336	178	6.3E+1	**0.01**	**0.06**	**6**	**150**	**10**	**4.2E+1**	**0.01**
b16	**0.12**	**23**	**223**	28	**2.4E+2**	**0.01**	0.17	24	239	**42**	3.1E+2	**0.01**
b17	**0.13**	24	183	38	2.7E+2	**0.01**	**0.13**	**17**	**145**	**12**	**1.5E+2**	**0.01**
b18	**0.18**	12	384	129	**1.1E+2**	**0.01**	0.27	**11**	**356**	**63**	**1.1E+2**	**0.01**
c01	0.27	58	181	41	**3.2E+2**	**0.01**	0.39	**48**	**153**	**5**	3.6E+2	**0.01**
c02	**0.98**	92	473	103	**1.2E+4**	**0.01**	1.01	**64**	**356**	**40**	4.3E+3	**0.01**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
c14	**19.86**	**19**	2212	494	**2.4E+3**	**0.98**	40.02	23	2581	**206**	3.7E+3	1.65
d01	**1.73**	133	428	70	4.5E+3	**0.01**	2.01	**96**	307	**5**	**2.3E+3**	**0.01**
d02	**1.81**	82	681	228	2.7E+3	**0.01**	2.44	**57**	483	**11**	**1.7E+3**	**0.01**
d03	**14.92**	35	4687	2231	1.4E+4	**0.4**	19.32	**20**	2762	**100**	**9.2E+3**	0.9
d04	**21.6**	32	6383	3114	**1.7E+4**	**0.63**	31.18	**19**	3687	**83**	2.3E+4	1.49
d05	**33.34**	29	13083	9892	3.4E+3	**1.1**	41.71	**13**	6209	**1469**	**2.6E+3**	2.95
d06	**8.12**	221	749	151	4.2E+4	**0.01**	14.29	**216**	716	**80**	**2.6E+4**	**0.01**
d07	**2.98**	92	549	166	**5.8E+3**	**0.01**	4.93	**85**	467	**71**	7.8E+3	**0.01**
d08	**66.44**	55	8324	2081	3.2E+5	**1.06**	84.39	**37**	5801	**698**	**2.1E+5**	2.09
d10	**71.64**	22	8609	1718	1.3E+4	**3.06**	86.61	**16**	5383	**436**	**9.2E+3**	5.07
d11	**15.12**	177	645	**194**	**1.4E+4**	**0.01**	30.19	**171**	665	215	1.9E+4	0.16
d12	**16.94**	**142**	763	**137**	9.7E+4	**0.11**	31.45	**161**	841	194	**6.5E+4**	0.18
Statistics	**1**	**1**	1	1	1	**1**	1.23	**0.76**	**0.73**	**0.21**	**0.82**	1.44

Reduction 24% rounds, 27% cuts, 79% duplicates, 18% condition number
Increase 23% time, 44% separation time

Min Steiner Tree - instances: i640, PUC

	Revised Standard Sep.					Revised Sequentially-Coordinated Sep.				
	Time	Rnds	Cuts	Dupl	SepTime	Time	Rnds	Cuts	Dupl	SepTime
i640-001	**1.3**	89	554	110	**0.01**	1.59	**74**	**417**	**28**	0.02
i640-011	**6**	90	**594**	**150**	0.06	9.59	**89**	598	156	**0.1**
i640-031	**2.04**	**73**	**498**	**67**	**0.02**	3.14	80	510	73	0.03
i640-041	**1002**	106	689	200	**9.42**	1012	**75**	**477**	**59**	13.46
i640-101	**7.36**	55	1210	179	**0.08**	9.11	**48**	**1035**	**35**	0.14
i640-131	**12.38**	60	1418	216	**0.14**	16.79	**55**	**1274**	**96**	0.25
i640-141	1021	40	955	26	**25.47**	**1015**	**23**	**546**	**23**	44.1
i640-201	**7.06**	48	1986	563	**0.11**	9.95	**44**	**1750**	**429**	0.19
i640-211	**1027**	80	3883	202	**3.3**	1038	**78**	**3791**	**138**	5.94
i640-231	**88.78**	**69**	**3400**	**437**	**0.51**	136.9	91	4482	1292	0.88
i640-301	**31.16**	35	5613	2715	**0.72**	39.81	**30**	**4638**	**970**	1.16
i640-311	**1107**	44	6978	180	**10.26**	1032	**39**	**6173**	**127**	15.92
i640-331	**433.1**	92	14747	5766	**2.1**	565.3	**84**	**13478**	**3446**	3.68
cc3-4p	17.51	175	1196	101	**0.01**	**12.43**	**161**	**1099**	**66**	**0.01**
cc3-4u	**32.73**	**235**	**1627**	88	**0.01**	46.91	237	1633	**44**	**0.01**
cc3-5p	624.9	399	4757	1877	**0.06**	**440.2**	**378**	**4518**	**1788**	0.07
cc3-5u	**1010**	343	4064	1607	**0.05**	**1010**	**293**	**3481**	**1467**	0.06
cc5-3p	1026	151	3890	304	**0.38**	**1024**	**130**	**3350**	**123**	0.55
cc5-3u	1035	139	3586	244	**0.39**	**1016**	**129**	**3319**	**226**	0.53
cc6-2p	**2.39**	78	804	149	**0.01**	2.69	**75**	**779**	**101**	**0.01**
cc6-2u	**2.62**	81	832	138	**0.01**	2.74	**77**	**826**	**92**	**0.01**
hc6p	**0.3**	18	571	232	**0.01**	0.43	**17**	**540**	**203**	0.02
hc6u	**0.25**	**18**	**538**	275	**0.01**	0.53	20	625	**220**	0.02
hc7p	**1.94**	22	1417	622	**0.07**	2.45	**17**	**1098**	**323**	0.11
hc7u	**3.45**	**26**	**1634**	909	**0.1**	9.34	27	1716	**741**	0.22
hc8p	**23.81**	**25**	**3269**	**1306**	**0.75**	35.21	**25**	**3269**	1361	1.18
hc8u	**648.9**	**54**	**6916**	**3455**	**1.76**	851.3	60	7687	3702	2.78
hc9p	**96.66**	**20**	**5298**	**1435**	**2.99**	139.9	21	5553	1730	5.13
hc9u	1200	30	7566	2727	**7.89**	**1170**	**26**	**6562**	**1801**	15.97
hc10p	**1073**	19	9654	1449	**26.07**	1202	**17**	**8632**	**907**	44.83
hc10u	**1695**	20	10119	2565	**23.62**	2028	**17**	**8603**	**1397**	49.67
Statistics	**1**	1	1	1	**1**	1.22	**0.92**	**0.92**	**0.68**	1.51

Reduction 8% rounds, 8% cuts, 32% duplicates

Increase 22% time, 51% separation time

Ongoing and further work

- Computional experiments suggest that cut coordination is an interesting feature
- Better rules for aggregating the previous cuts into $\bar{\alpha}^t$, e.g., adaptive

  Weighted average with $\bar{\alpha}^t := 0.7\bar{\alpha}^{t-1} + 0.3\alpha^t$: further reduction of 3% rounds, 7% duplicates, and 5% time
- Investigate other diversity measures
- Cut coordination among different families
- Extension to cuts with not only 0-1 coefficients
- Cut coordination in branch-and-cut (node-by-node or coordinated pool)