Column Generation for Extended Formulations

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Abstract

Working in an extended variable space allows one to develop tight reformulations for mixed integer programs. However, the size of the extended formulation grows rapidly too large for a direct treatment by a MIP-solver. Then, one can use projection tools to derive valid inequalities for the original formulation and implement a cutting plane approach. Or, one can approximate the reformulation, using techniques such as variable aggregation or by reformulating a submodel only. Such approaches result in outer approximation of the intended extended formulation. The alternative considered here is an inner approximation obtained by generating dynamically the variables of the extended formulation. It assumes that the extended formulation stems from a decomposition principle: a subproblem admits an extended formulation from which an extended formulation for the original problem can be derived. Then, one can implement column generation for the extended formulation of the original problem by transposing the equivalent procedure for the Dantzig-Wolfe reformulation. Pricing subproblem solutions are expressed in the variables of the extended formulation and added to the current restricted version of the extended formulation along with the subproblem constraints that are active for the subproblem solution. The paper reviews the applications of the literature of such "column-and-row generation" procedure and analyses this approach's potential benefits compared to a standard column generation approach. Numerical experiments highlight a key observation: lifting pricing problem solutions in the space of the extended formulation permits their recombination into new subproblem solutions and results in faster convergence.

Keywords: Extended formulations for MIP, Column-and-Row Generation, Stabilization.